$$\overline{\cos\theta} = \frac{\int F(\Omega)\cos\theta d\Omega}{\int F(\Omega)\, d\Omega}, \tag{3.13}$$

in terms of an unknown distribution function of the magnetization vector directions throughout the solid angle. A first harmonic assumption for the distribution function is that $F(\Omega)$ is uniform throughout the solid angle defined by the extreme angles from the <100> problem and the <111> problem. These were

$$\cos\theta_1 = -\frac{M_s}{2b_1 e} H_e \quad \text{and} \quad \cos\theta_2 = -\frac{M_s}{2b_2 e} H_e.$$

$F(\Omega)$ is zero otherwise. This first harmonic approximation gives for the average value

$$\overline{\cos\theta} = \frac{\int_{\theta_1}^{\theta_2} \cos\theta\sin\theta d\theta}{\int_{\theta_1}^{\theta_2} \sin\theta d\theta} = \frac{\int_{x_2}^{x_1} x\, dx}{\int_{x_2}^{x_1} dx}$$

where $x = \cos\theta$. A problem occurs when $\cos\theta_1$ is unity at which point the first grains reach saturation. To freeze the upper limit of integration artificially constrains the distribution function. This problem can be circumvented by allowing the upper limit to continue but demanding that the respective contribution to $\cos\theta$ be unity. This gives

$$\overline{\cos\theta} = \begin{cases} \dfrac{\int_{x_2}^{x_1} x\,dx}{\int_{x_2}^{x_1} dx} & \text{for} \quad x_1 \le 1 \\[2ex] \dfrac{\int_{x_2}^{1} x\,dx + \int_{1}^{x_1} dx}{\int_{x_2}^{x_1} dx} & \text{for} \quad x_1 \ge 1. \end{cases}$$

Performing the integration gives

$$\overline{\cos\theta} = \begin{cases} \frac{1}{2}(x_1 + x_2) & \text{for} \quad x_1 \le 1 \\[1ex] \dfrac{\frac{1}{2}(x_2^2 - 2x_1 + 1)}{x_2 - x_1} & \text{for} \quad x_1 \ge 1. \end{cases}$$

This will be expressed in the final form by

$$\frac{M}{M_s} = \begin{cases} \frac{1}{2}(\eta_1 + \eta_2)H_e & \text{for} \quad \eta_1 H_e \le 1 \\[1ex] \dfrac{\frac{1}{2}(\eta_2^2 H_e^2 - 2\eta_1 H_e + 1)}{(\eta_2 - \eta_1)H_e} & \text{for} \quad \eta_1 H_e \ge 1. \end{cases} \tag{3.14}$$

where

$$\eta_1 = -\frac{M_s}{2b_1 e} \quad \text{and} \quad \eta_2 = -\frac{M_s}{2b_2 e}. \tag{3.15}$$

The solution is $M/M_s = 1$ above the magnetic field for which $\eta_2 H_e = 1$.